Graphs of functions and Travel graphs

Compiled by : Mustafa Asif

24. Graphs in practical situations	• interpret and use graphs in practical situations including travel graphs and conversion graphs • draw graphs from given data • apply the idea of rate of change to easy kinematics involving distance–time and speed–time graphs, acceleration and deceleration • calculate distance travelled as area under a linear speed–time graph	
25. Graphs of functions	• construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant, and $n = -2, -1, 0, 1, 2, 3$, and simple sums of not more than three of these and for functions of the form ka^x where a is a positive integer • interpret graphs of linear, quadratic, cubic, reciprocal and exponential functions • solve associated equations approximately by graphical methods • estimate gradients of curves by drawing tangents	

Videos for understanding

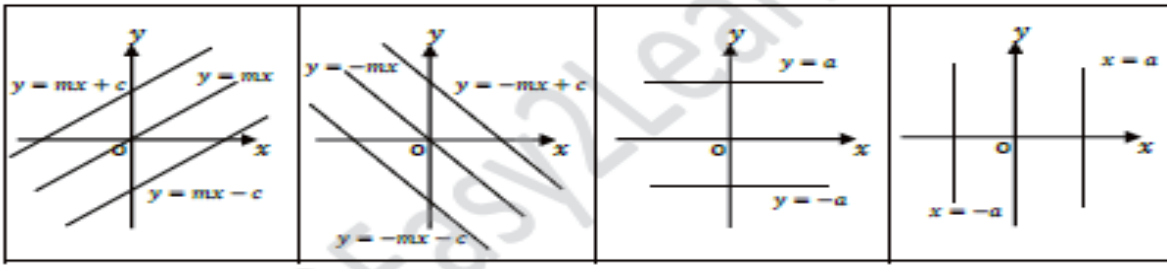
https://www.youtube.com/watch?v=eCEXUw-mtdE&list=PLjK050gbQMRKZSq_HtXJBXqKQVaV4rzb3&index=2

https://www.youtube.com/watch?v=LJctqrA9jhU

https://www.youtube.com/watch?v=7C3f-sYMNCU

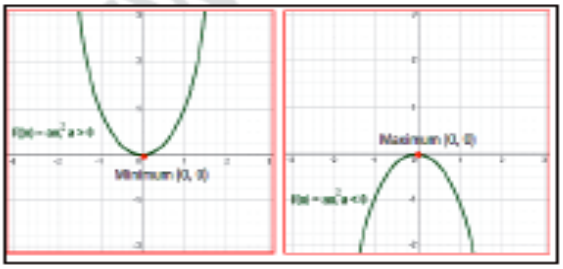
Graphs of Functions and Graphical Solutions

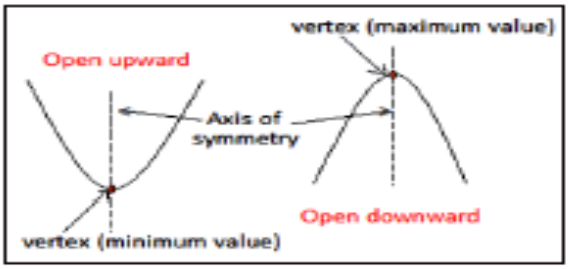
Graphs of Linear Function: $f(x) = mx + c$



Graphs of Quadratic Function : $f(x) = ax^2 + bx + c$

Graphing Quadratic Functions in Vertex Form : A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in vertex form as $y = a(x - h)^2 + k$, where the vertex of the graph is (h, k) and the axis of symmetry is $x = h$.





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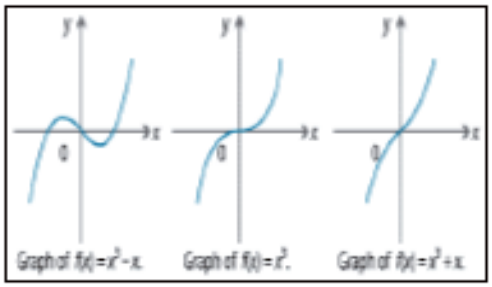
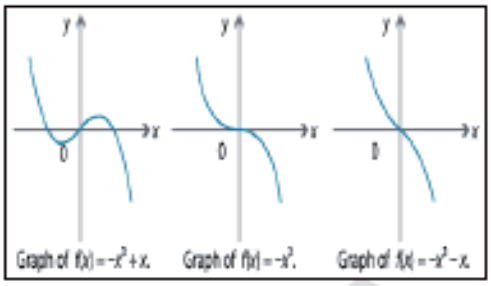
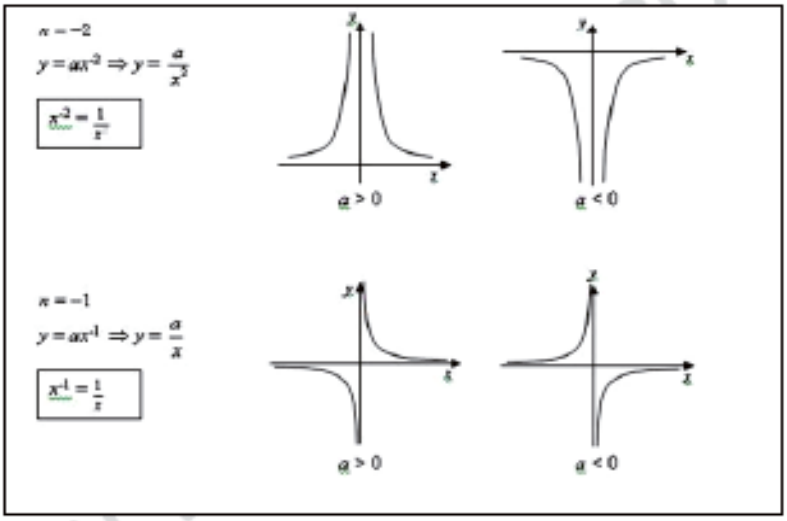
www.2Easy2Learn.Com

Graphs of Cubic Functions: $f(x) = ax^3 + bx^2 + cx + d$

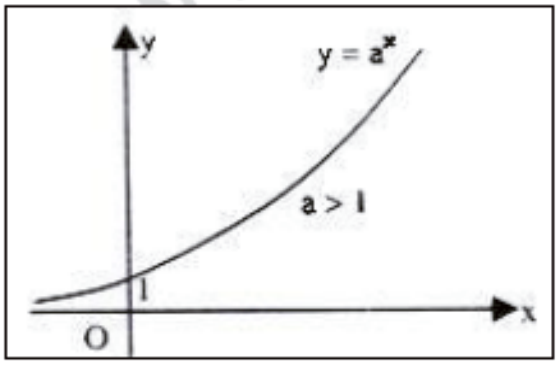


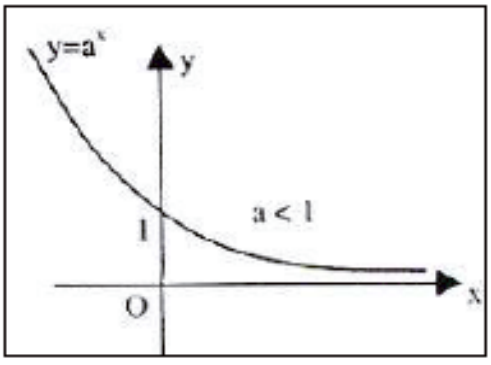


Graphs of Reciprocal Functions:



Graphs of Exponential Functions





Graphs in Practical Situation and Travel Graphs

Key Points

Speed: Speed can be defined as the distance covered by a moving object in unit time taken. SI unit of speed is m/s or ms^{-1}. Speed is a scalar quantity.

Uniform Speed: An object is said to be moving with uniform speed if it covers equal distances in equal intervals of time.

Variable Speed: An object is said to be moving with variable speed or non-uniform speed if it covers equal distances in unequal intervals of time or vice-versa.

Instantaneous Speed: The speed that the body possesses at a particular instant of time, is called instantaneous speed.

Average Speed: When we travel in a vehicle the speed of the vehicle changes from time to time depending upon the conditions existing on the road. In such a situation, the speed is calculated by taking the ratio of the total distance traveled by the vehicle to the total time taken for the journey. This is called the average speed

$$\text{Average Speed} = \frac{\text{Total Distance Traveled}}{\text{Total Time Taken}}$$

Acceleration: Acceleration is defined as the rate of change of velocity of a moving body with time. This change could be a change in the speed of the object or its direction of motion or both. Let an object moving with an initial velocity 'u' attain a final velocity 'v' in time 't', then acceleration 'a' produced in the object is

Acceleration = Rate of change of velocity with time

$$Acceleration = \frac{Change\ in\ velocity}{Time} \qquad a = \frac{v-u}{t}$$

The SI unit of velocity is m/s and time is s

∴ **SI unit of acceleration is** $\frac{\frac{m}{s}}{s} = \frac{m}{s^2}$ **Acceleration is a vector quantity.**

Positive Acceleration : If the velocity of an object increases then the object is said to be moving with positive acceleration.

Example: A ball rolling down on an inclined plane.

Negative Acceleration : If the velocity of an object decreases then the object is said to be moving with negative acceleration. Negative acceleration is also known as retardation or deceleration.

Examples: A ball moving up an inclined plane or a ball thrown vertically upwards is moving with a negative acceleration as the velocity decreases with time.

Graphs of functions and Travel graphs

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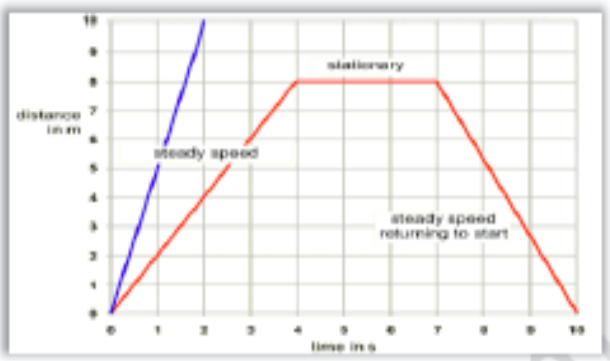
Zero Acceleration: If the change in velocity is zero, i.e., either the object is at rest or moving with uniform velocity, then the object is said to have zero acceleration.

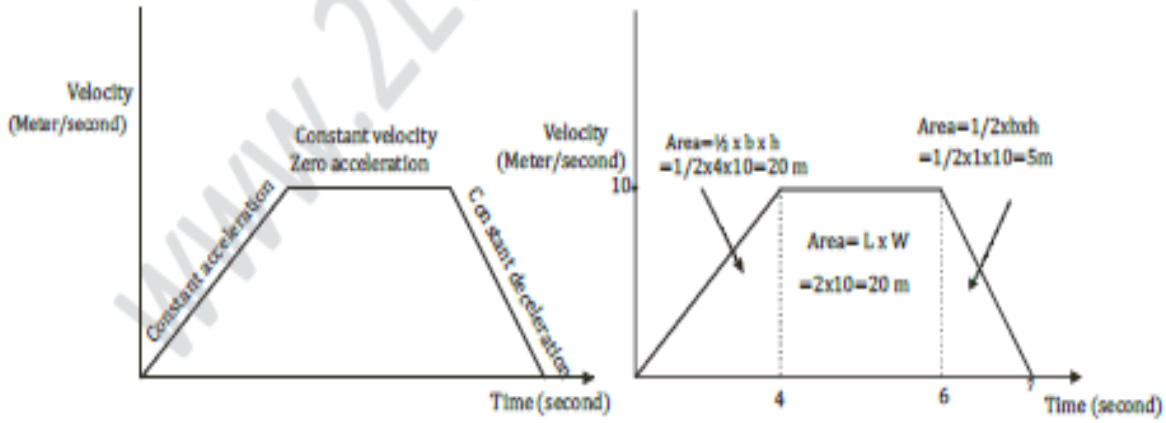
The Distance-Time Graph



- Vertical coordinate shows distance.
- Horizontal coordinates shows time.
- Slope (gradient) shows speed.
- Straight line segments indicate constant speed.
- Graph getting steeper indicates getting faster, graph getting shallower indicates slowing down
- Level parts (horizontal line) indicate stopping

Speed-time graphs

- Vertical coordinate shows Speed.
- Slope (gradient) shows constant acceleration.
- Horizontal line segments indicate constant speed (acceleration is zero)..
- Area under the curve shows distance.
- Moving away from the horizontal axis indicates getting faster, moving towards the horizontal axis indicates getting slower.



- Graph above the horizontal axis indicates moving forward, graph below the horizontal axis indicates moving backward.
- Points on the horizontal axis indicate stopping.

Graphs of functions and Travel graphs

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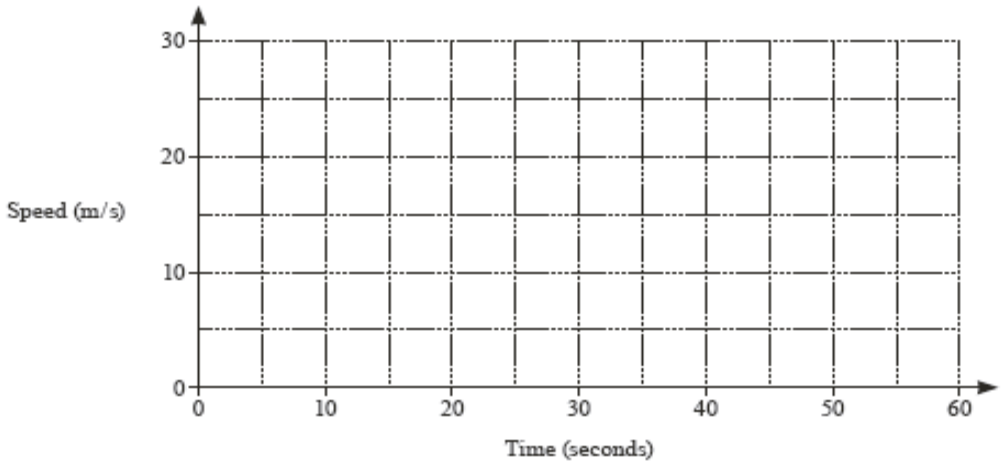
1 A car starts a journey from rest.
It moves with constant acceleration for 20 seconds until it reaches a speed of 15 m/s.
It then moves at a constant speed of 15 m/s for 40 seconds. M/J19/12/9

(a) On the grid, draw the speed–time graph for the car's journey.



[2]

(b) Calculate the acceleration of the car in the first 20 seconds of the journey.

.....m/s^2 [1]

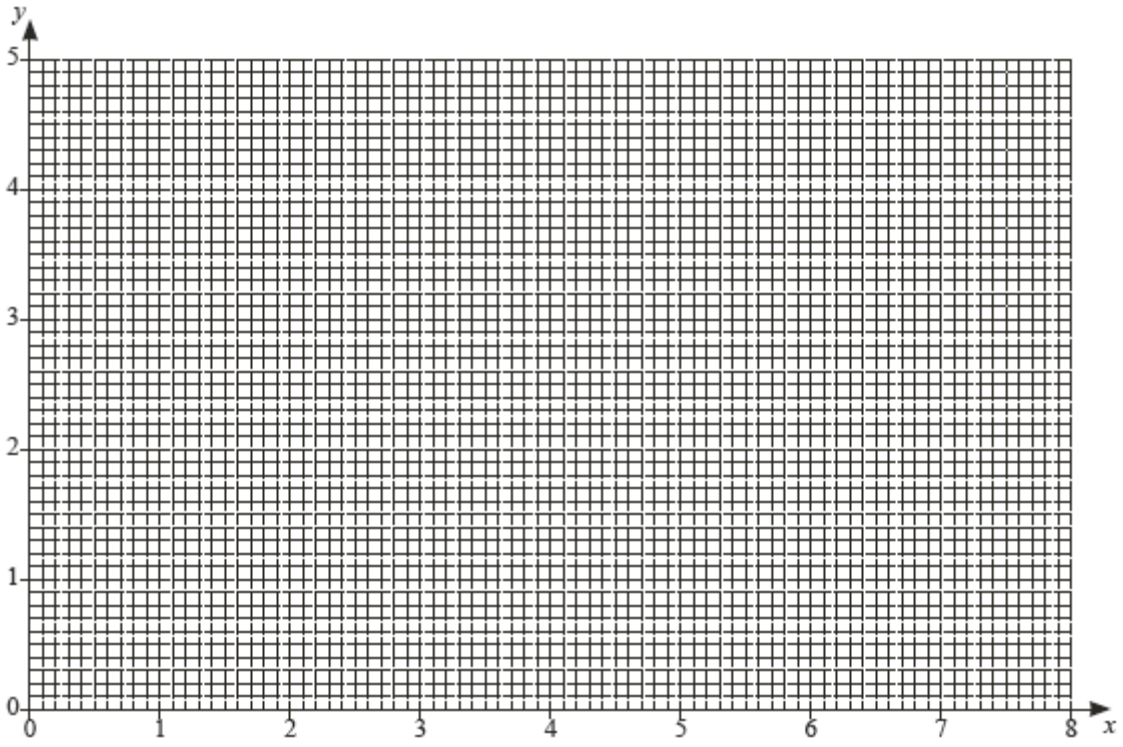
2 The table shows some values for $y = 1 + \frac{2}{x}$, given correct to 2 decimal places where appropriate.

x	0.5	1	2	3	4	5	6	7	8
y	5	3	2	1.67	1.5	1.4	1.33	1.29	

(a) Complete the table. [1]

M/J19/22/7

(b) Draw the graph of $y = 1 + \frac{2}{x}$ for $0.5 \leqslant x \leqslant 8$.



[2]

(c) The line L crosses the graph of $y = 1 + \frac{2}{x}$ at $x = 2$ and $x = 5$.

Find the equation of L.

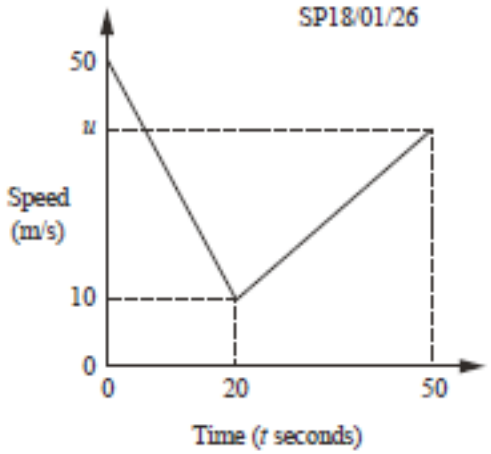
..... [3]

(d) A line with gradient $-\frac{1}{3}$ crosses the graph of $y = 1 + \frac{2}{x}$ when $x = 1$ and when $x = k$.

By drawing a suitable line on your grid, find k.

$k =$ ..... [2]

3 The diagram is the speed–time graph of part of a train's journey.



The train slows down uniformly from a speed of 50 m/s to a speed of 10 m/s in a time of 20 seconds.

During the next 30 seconds, it accelerates uniformly to a speed of u m/s.

(a) Calculate the deceleration from $t = 0$ to $t = 20$.

Answer ..... m/s^2 [1]

(b) Calculate the speed of the train when $t = 15$.

Answer ..... m/s [1]

(c) Calculate the distance travelled by the train from $t = 0$ to $t = 20$.

Answer ..... m [1]

(d) The size of the acceleration is half the size of the deceleration.
Find the value of u.

Answer $u =$ ..... [1]

4 (a) The table shows some values of x and the corresponding values of y for $y = 2x^3 - 3x^2 + 5$.

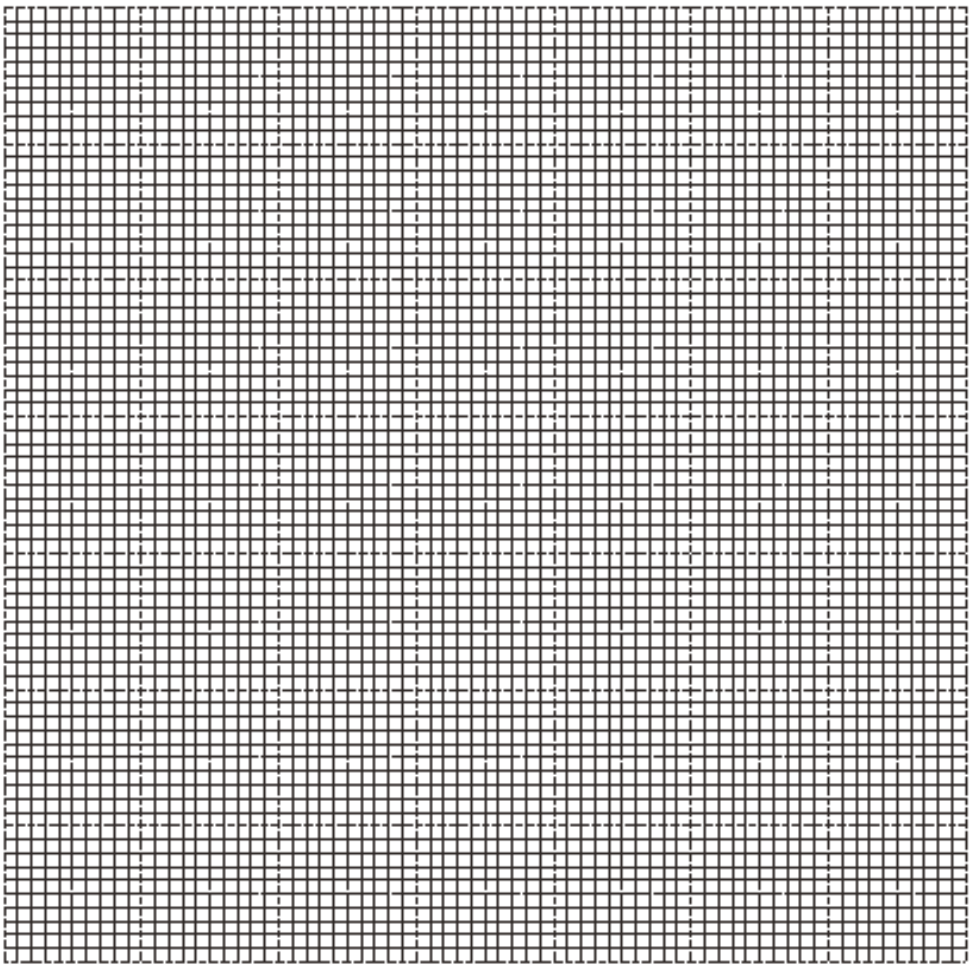
x	–1.5	–1	–0.5	0	0.5	1	1.5	2
y		0	4	5	4.5	4	5	9

(i) Complete the table. [1]

(ii) Using a scale of 4 cm to represent 1 unit, draw a horizontal x-axis for $-1.5 \leqslant x \leqslant 2$.
Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for $-10 \leqslant y \leqslant 10$.

SP18/02/6(a)

Draw the graph of $y = 2x^3 - 3x^2 + 5$ for $-1.5 \leqslant x \leqslant 2$.



[3]

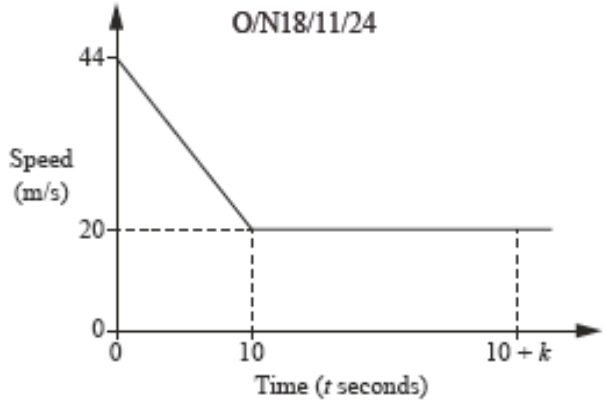
(iii) Use your graph to estimate the gradient of the curve when $x = 1.5$.

Answer ..... [2]

(iv) By drawing a suitable line on your graph, find the solution of the equation $2x^3 - 3x^2 + 4 = 0$.

Answer $x =$ ..... [2]

5 The diagram is the speed–time graph of part of a train's journey.



The train slows down uniformly from a speed of 44 m/s to a speed of 20 m/s in a time of 10 seconds. It then continues at a constant speed of 20 m/s.

(a) Find the deceleration when $t = 5$.

Answer ..... m/s^2 [1]

(b) Find the speed when $t = 5$.

Answer ..... m/s [1]

(c) The distance travelled from $t = 0$ to $t = 10$ is equal to the distance travelled from $t = 10$ to $t = 10 + k$.

Find k.

Answer $k =$ ..... [3]

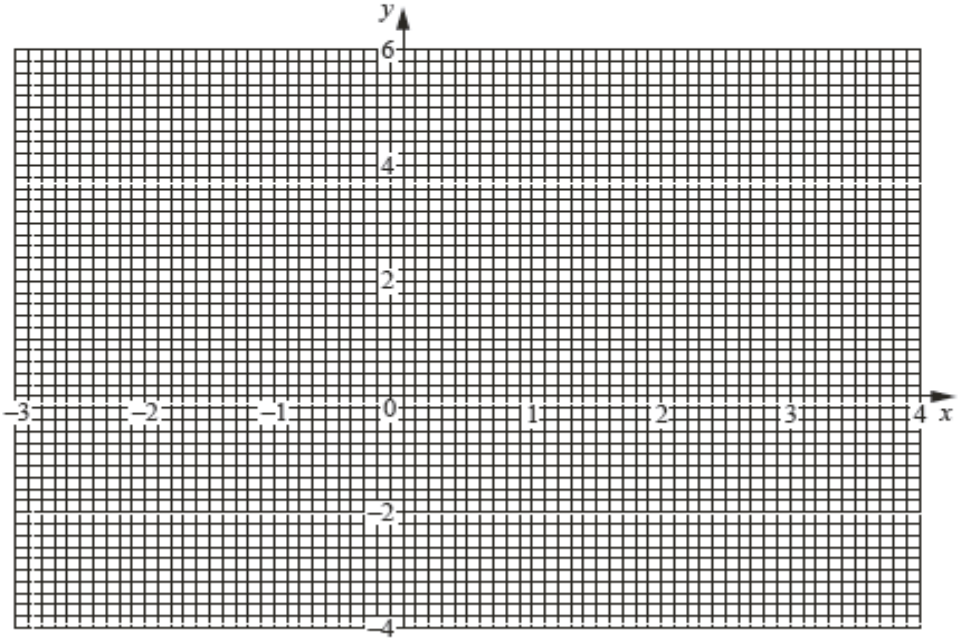
6 **(a)** Complete the table for $y = \frac{x}{5}(6 + 2x - x^2)$.

x	–3	–2	–1	0	1	2	3	4
y	5.4	0.8	–0.6	0	1.4	2.4	1.8	

O/N18/21/5

[1]

(b) Draw the graph of $y = \frac{x}{5}(6 + 2x - x^2)$ for $-3 \leqslant x \leqslant 4$.



[3]

(c) By drawing a tangent, estimate the gradient of $y = \frac{x}{5}(6 + 2x - x^2)$ at (–2, 0.8).

Answer ..... [2]

(d) Use your graph to solve the equation $x(6 + 2x - x^2) = 10$.

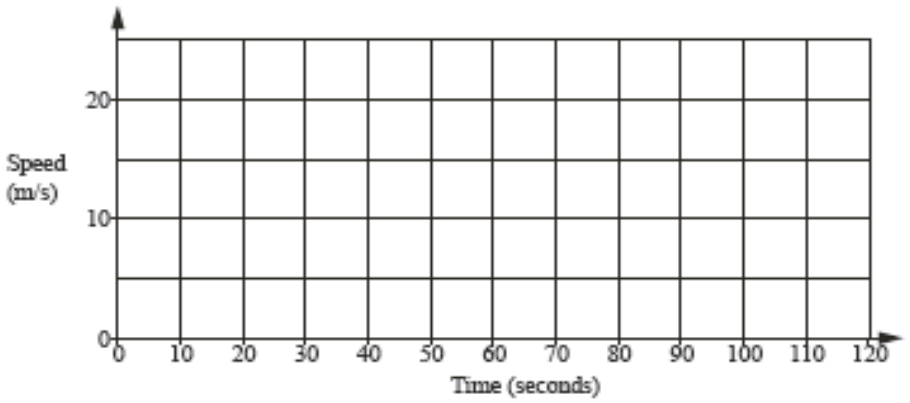
Answer ..... [3]

7 A train travels between two stations, starting and finishing at rest.

For this journey it
- accelerates uniformly, from rest, for the first 30 seconds until it reaches a speed of 20 m/s
- travels at a constant speed of 20 m/s for the next 60 seconds
- slows down uniformly for the last 20 seconds until it stops.

O/N18/12/24

(a) On the grid, draw the speed–time graph for this journey.



[2]

(b) Calculate the distance between the stations.

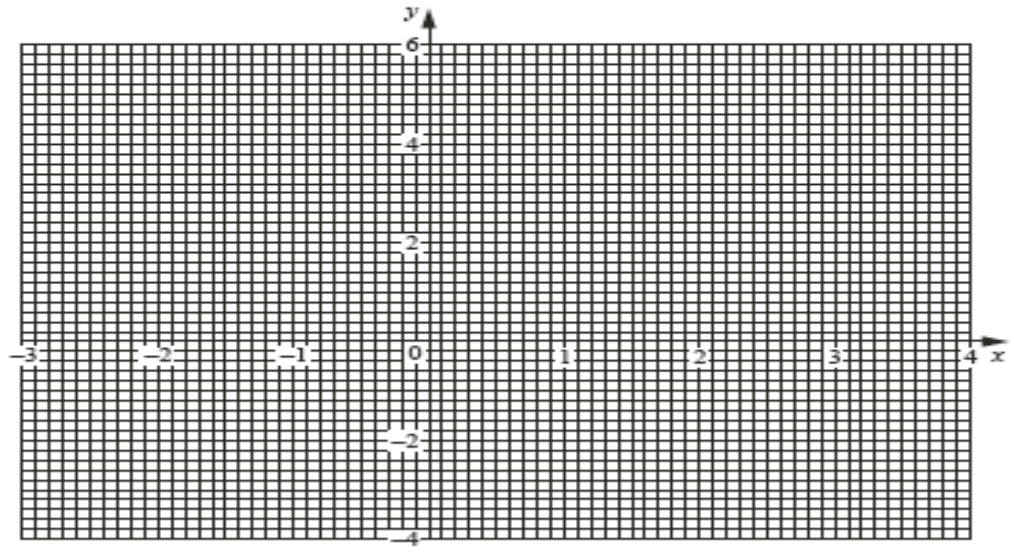
Answer ..... m [2]

8 (a) Complete the table for $y = \frac{x}{5}(6 + 2x - x^2)$. O/N18/21/5

x	–3	–2	–1	0	1	2	3	4
y	5.4	0.8	–0.6	0	1.4	2.4	1.8	

[1]

(b) Draw the graph of $y = \frac{x}{5}(6 + 2x - x^2)$ for $-3 \leqslant x \leqslant 4$.



[3]

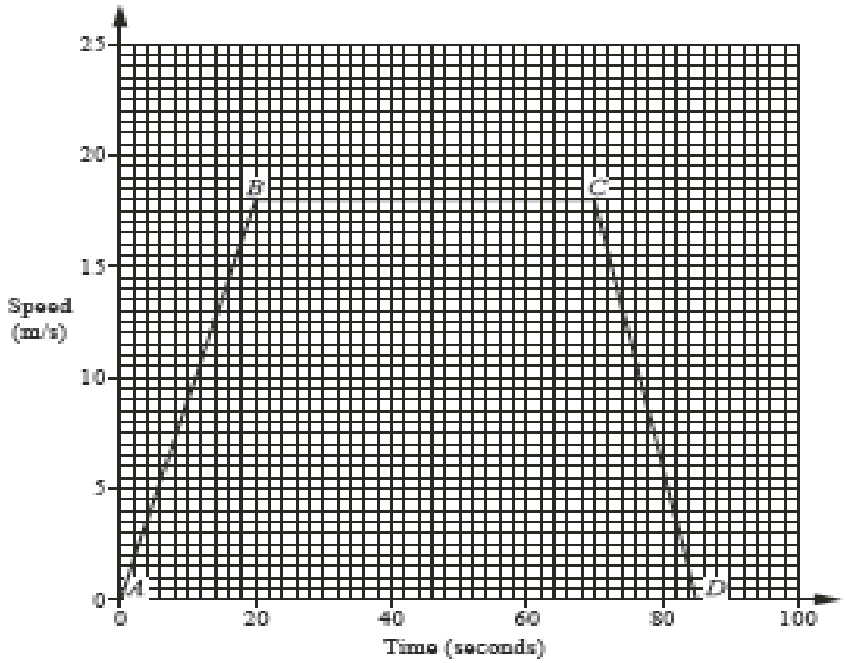
(c) By drawing a tangent, estimate the gradient of $y = \frac{x}{5}(6 + 2x - x^2)$ at (–2, 0.8).

Answer ..... [2]

(d) Use your graph to solve the equation $x(6 + 2x - x^2) = 10$.

Answer ..... [3]

9 M/J18/11/18



The speed-time graph shows the motion of a car.

(a) Describe **fully** the motion of the car represented by each of the lines *AB*, *BC* and *CD* on the graph.

AB has been done for you.

AB Accelerates for the first 20 s at 0.9 m/s^2.

BC .....

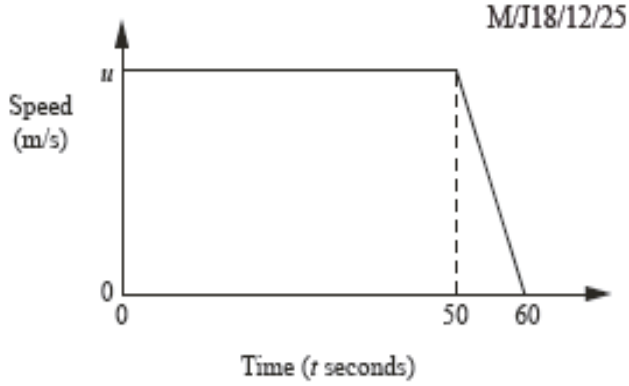
CD ..... [3]

(b) Find the total distance travelled by the car during this motion.

Answer ..... m [2]

10 The diagram is the speed–time graph for 60 seconds of a train's journey.
At the beginning of this part of the journey the train is travelling at u m/s.

M/J18/12/25



Giving each answer in its simplest form, find expressions in terms of u, for

(a) the deceleration for the last 10 seconds,

Answer ..... m/s^2 [1]

(b) the speed when $t = 55$,

Answer ..... m/s [1]

(c) the distance travelled during these 60 seconds.

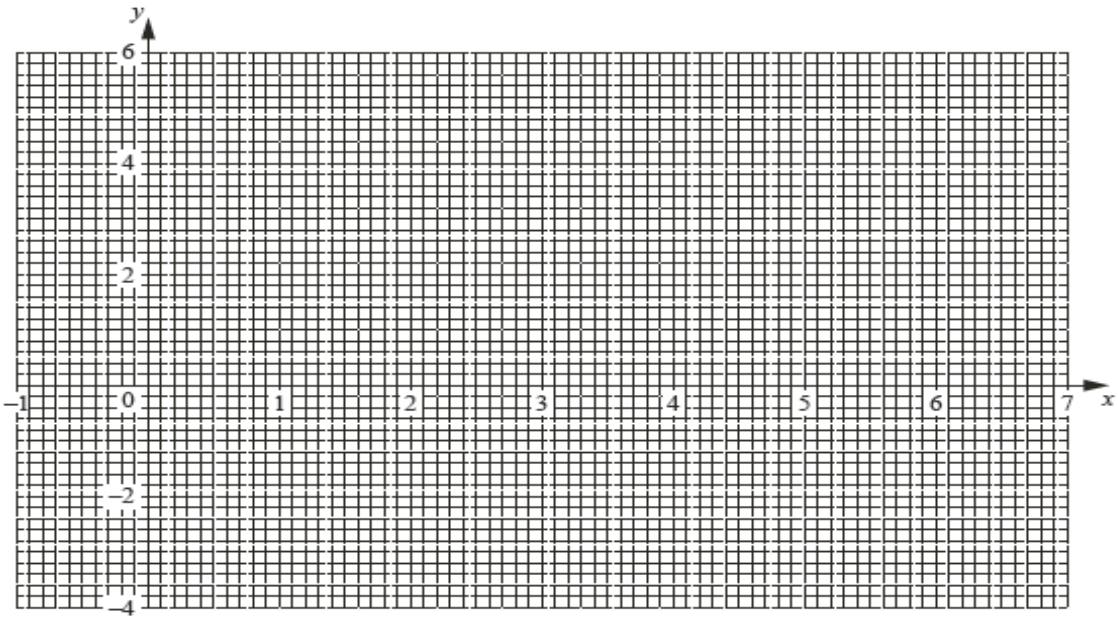
Answer ..... m [2]

11 **(a)** Complete the table for $y = \frac{x^2}{2} - 3x + 2$. M/J18/22/6

x	−1	0	1	2	3	4	5	6	7
y		2	−0.5	−2	−2.5	−2	−0.5	2	

[1]

(b) Draw the graph of $y = \frac{x^2}{2} - 3x + 2$ for $-1 \leqslant x \leqslant 7$.



[3]

(c) By drawing a tangent, estimate the gradient of the curve at $x = 1.5$.

Answer ..... [2]

(d) Complete these inequalities to describe the range of values of x where $y \geqslant 0$.

Answer $x \leqslant$ .....

$x \geqslant$ ..... [2]

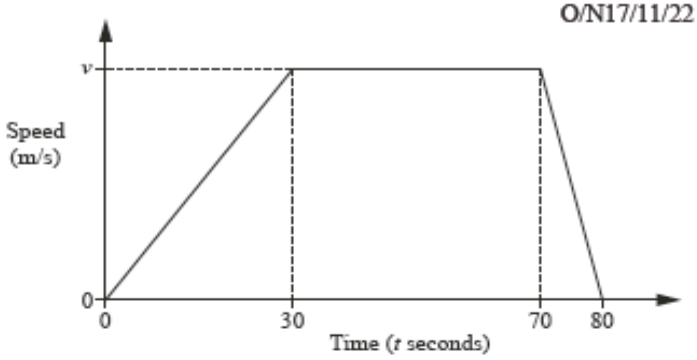
(e) **(i)** On the same grid, draw the line $4y + 3x = 12$. [2]

(ii) The x-coordinates of the points of intersection of this line and the curve are the solutions of the equation $2x^2 + Ax + B = 0$.

Find the value of A and the value of B.

12 The diagram is the speed-time graph of a train which travels between two stations.

O/N17/11/22



(a) Find an expression, in terms of v, for the retardation of the train.

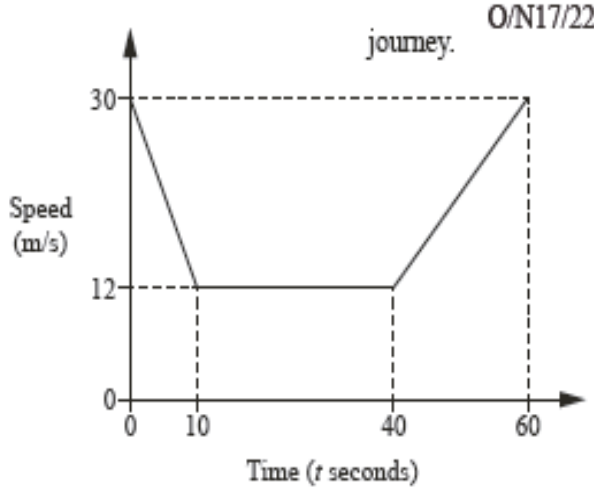
Answer ..... m/s^2 [1]

(b) The distance between the two stations is 1.2 km.

Find v.

Answer $v =$ ..... [3]

13 The diagram is the speed-time graph of part of a train's journey. O/N17/22/24



(a) Calculate the speed when $t = 5$.

Answer ..... m/s [1]

(b) Calculate the acceleration.

Answer ..... m/s^2 [1]

(c) Calculate the distance travelled from $t = 40$ to $t = 60$.

Answer ..... m [2]

14 (a) (i) The points (4, −3) and (0, 5) lie on the line L.
Find the equation of line L. O/N17/21/7

Answer ..... [2]

(ii) The line M is parallel to line L and passes through the point (−2, 3).
Find the equation of line M.

Answer ..... [2]

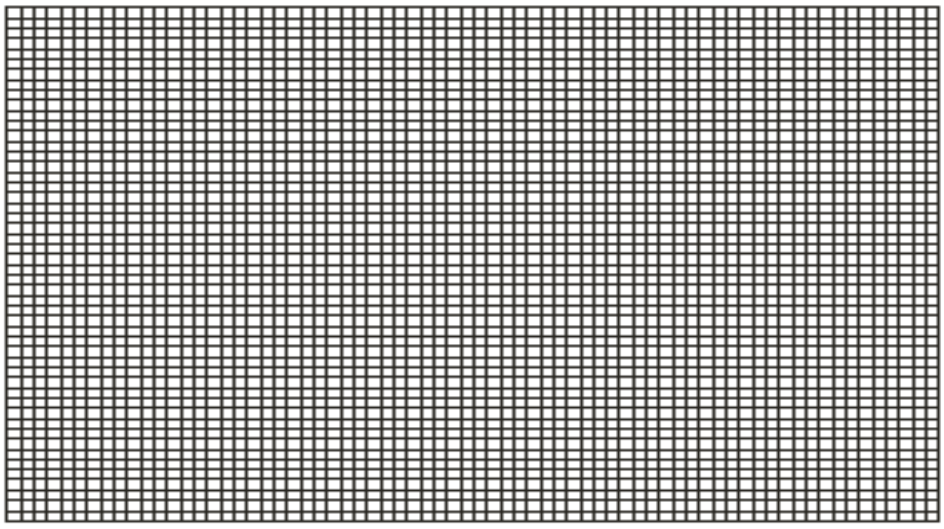
(b) The table below shows some values of x and the corresponding values of y for $y = x + \frac{3}{x} - 3$.

x	0.5	1	1.5	2	3	4	5	6
y	3.5	1	0.5	0.5	1	1.75	2.6	

(i) Complete the table. [1]

(ii) Using a scale of 2 cm to 1 unit on both axes, draw a horizontal x-axis for $0 \leqslant x \leqslant 7$ and a vertical y-axis for $0 \leqslant y \leqslant 4$.

Draw the graph of $y = x + \frac{3}{x} - 3$ for $0.5 \leqslant x \leqslant 6$.



[3]

(iii) By drawing a tangent, estimate the gradient of the curve at (1, 1).

Answer ..... [2]

(iv) Use your graph to solve the equation $x + \frac{3}{x} = 5$.

Answer $x =$ ..... or ..... [2]

15(a) The variables x and y are connected by the equation $y = 3 + x - \frac{x^2}{2}$.

Some corresponding values of x and y are given in the table below.

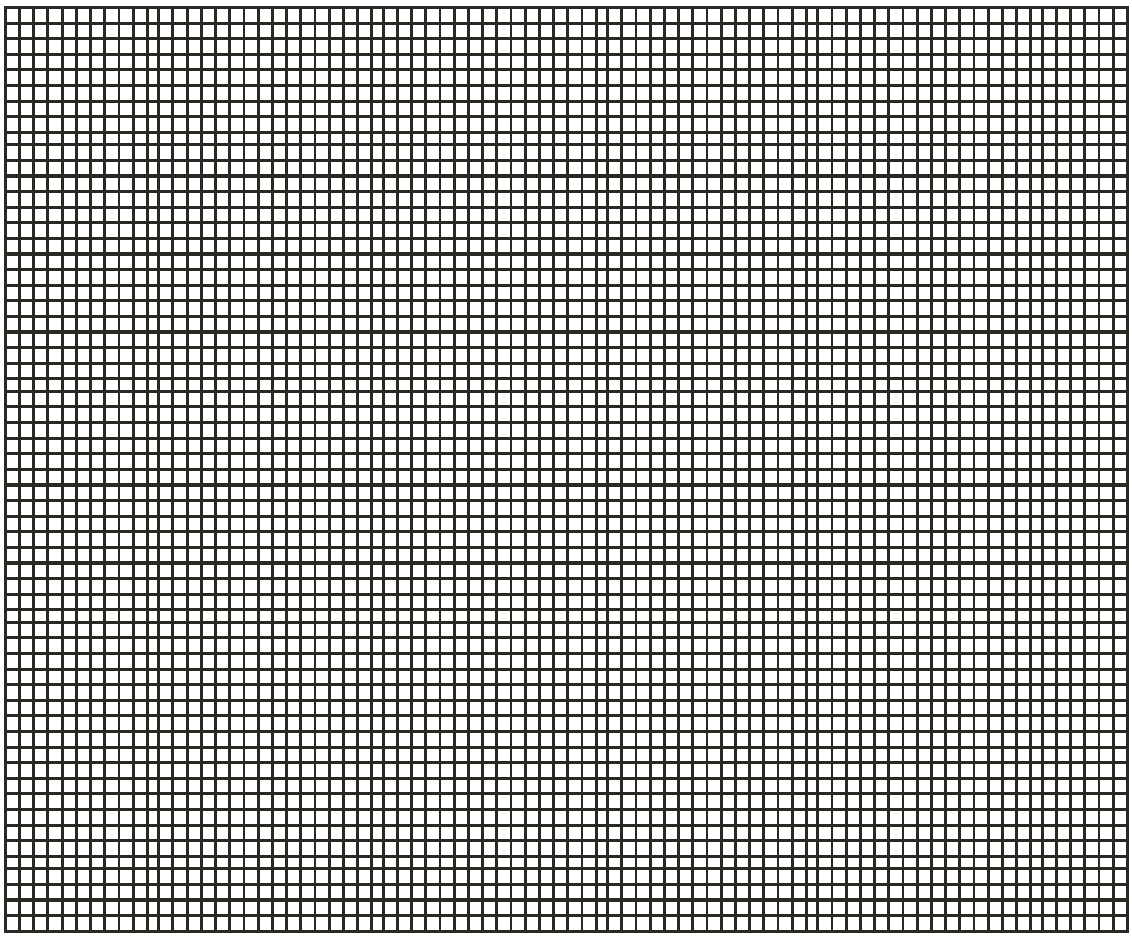
x	−3	−2	−1	0	1	2	3	4	5
y		−1	1.5	3	3.5	3	1.5	−1	

O/N17/22/7

(i) Complete the table. [1]

(ii) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-3 \leqslant x \leqslant 5$.
Using a scale of 1 cm to 1 unit, draw a vertical y-axis for $-5 \leqslant y \leqslant 5$.

Draw the graph of $y = 3 + x - \frac{x^2}{2}$ for $-3 \leqslant x \leqslant 5$.



[3]

(iii) By drawing a tangent, estimate the gradient of the curve at (3, 1.5).

Answer ..... [2]

(iv) The points of intersection of the graph of $y = 3 + x - \frac{x^2}{2}$ and the line $y = k$ are the solutions of the equation $10 + 2x - x^2 = 0$.

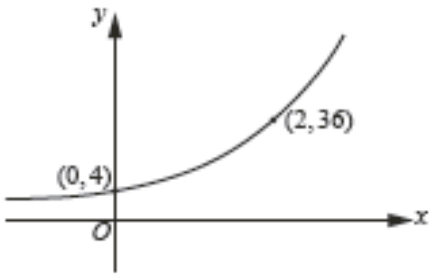
(a) Find the value of k.

Answer ..... [1]

(b) By drawing the line $y = k$ on your graph, find the solutions of the equation $10 + 2x - x^2 = 0$.

Answer ..... [2]

(b) This is a sketch of the graph of $y = pa^x$, where $a > 0$.
The graph passes through the points (0, 4) and (2, 36).



(i) Write down the value of p.

Answer ..... [1]

(ii) Find the value of a.

Answer ..... [1]

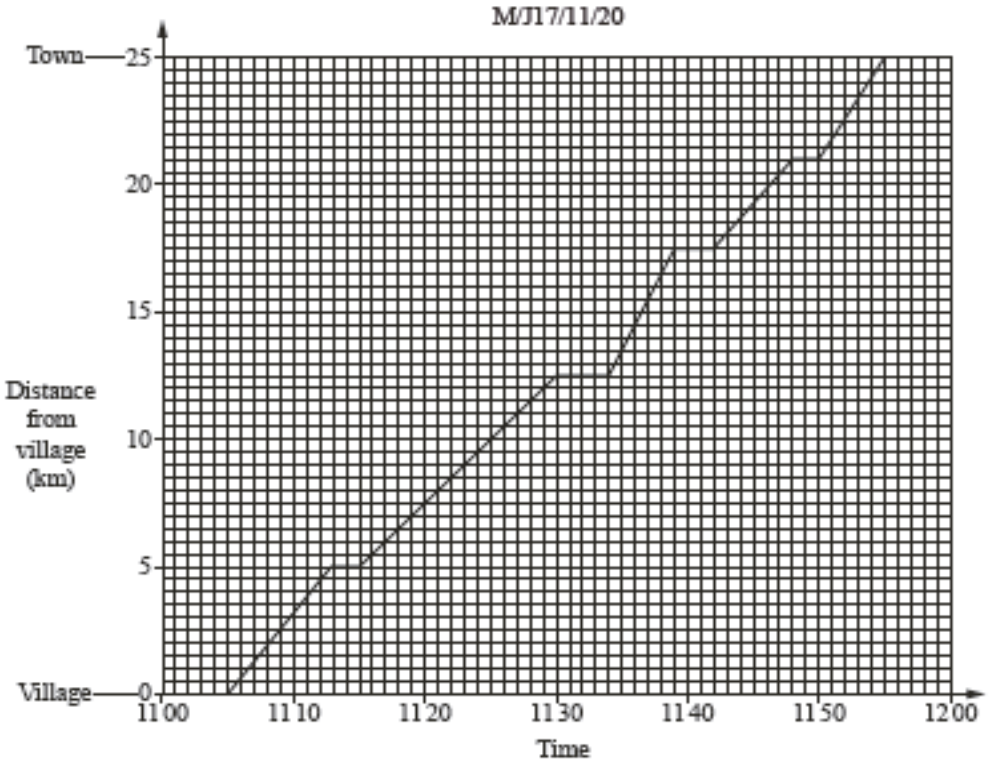
(iii) The graph passes through the point $(4, q)$.

Find the value of q.

Answer ..... [1]

16

M/J17/11/20



The distance-time graph shows the journey of a red bus travelling from a village to a town.

(a) Find the total length of time for which the bus is stopped during the journey.

Answer .....minutes [1]

(b) Find the average speed of the bus over the **whole** journey from the village to the town.

Answer .....km/h [1]

(c) A yellow bus leaves the **town** at 11 25 and travels non-stop along the same road to the village at a constant speed of 50 km/h.

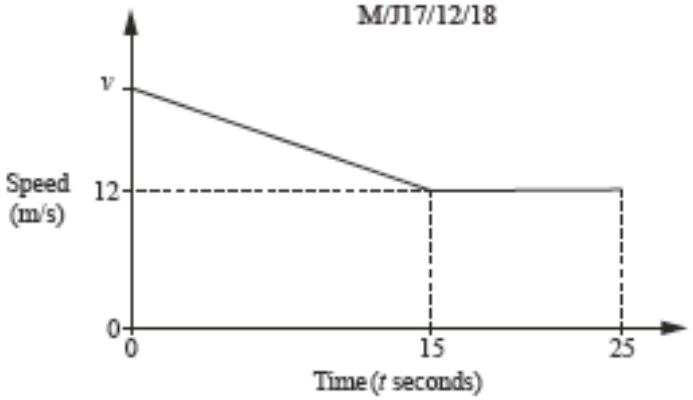
(i) On the graph draw the distance-time graph for the yellow bus. [1]

(ii) At what time does the yellow bus meet the red bus?

Answer ..... [1]

17 The diagram is the speed-time graph for 25 seconds of a car's journey.

M/J17/12/18



The car slows down uniformly from a speed of v m/s to a speed of 12 m/s in 15 seconds.
It then travels at constant speed for a further 10 seconds.

(a) The retardation of the car is $0.4\,m/s^2$.

Calculate the value of v.

Answer $v =$ ..... [2]

(b) Calculate the distance travelled by the car from $t = 0$ to $t = 25$.

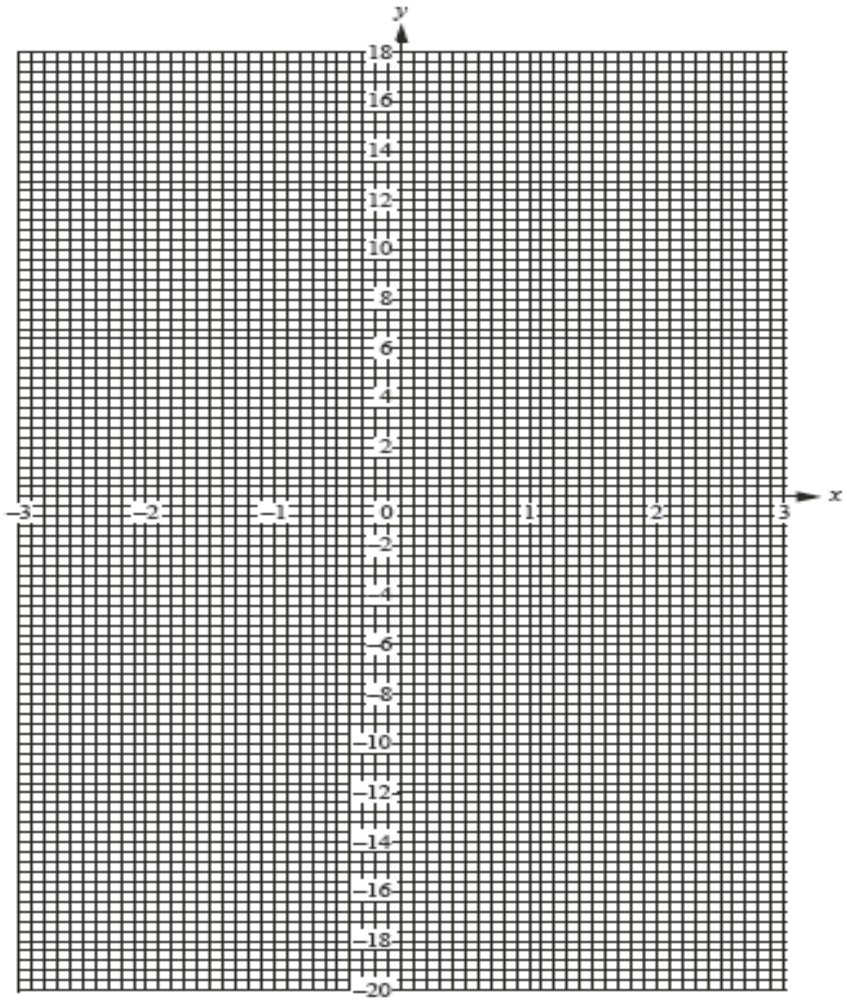
Answer ..... m [2]

18 The table below is for $y = x^3 - 3x - 1$.

x	–3	–2	–1	0	1	2	3
y	–19	–3	1	–1	–3	1	

(a) Complete the table. M/J17/21/5 [1]

(b) On the grid, draw the graph of $y = x^3 - 3x - 1$.



[3]

(c) Use your graph to solve $x^3 - 3x - 1 = 0$.

Answer $x =$ ..... [2]

(d) Use your graph to estimate the gradient of the curve when $x = -1.5$.

Answer ..... [2]

(e) (i) On the grid draw the graph of $y = 4x + 3$. [1]

(ii) The line $y = 4x + 3$ and the curve $y = x^3 - 3x - 1$ can be used to solve the equation $x^3 = ax + b$.

Find the values of a and b.

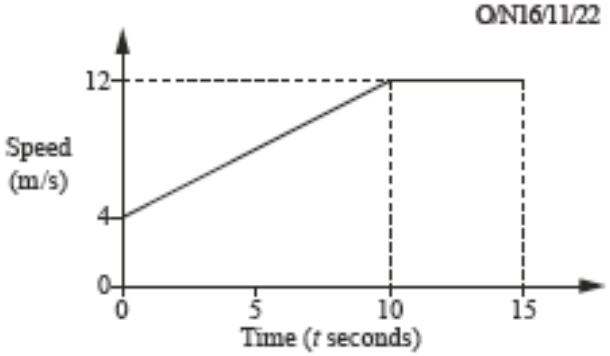
Answer $a =$ ..... $b =$ ..... [2]

(iii) Use your graph to find one of the **negative** solutions of $x^3 = ax + b$.

Answer $x =$ ..... [1]

19 The diagram is the speed-time graph of part of a car's journey.

O/N16/11/22



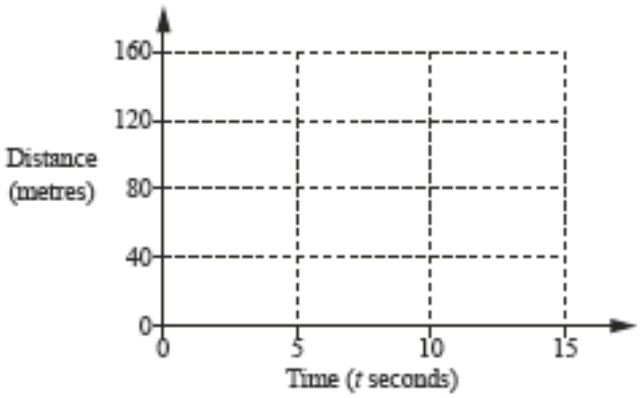
(a) Find the speed when $t = 8$.

Answer ..... m/s [2]

(b) Find the distance travelled by the car from $t = 0$ to $t = 10$.

Answer ..... m [2]

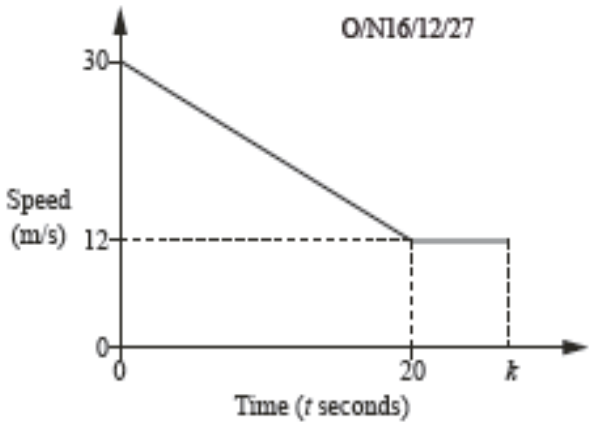
(c) On the diagram below sketch the distance-time graph for $t = 0$ to $t = 15$.



[2]

20

O/N16/12/27



The diagram shows the speed-time graph of a car which slows down from 30 m/s to 12 m/s in 20 seconds, and then continues at a speed of 12 m/s.

(a) Find the retardation when $t = 10$.

Answer ..... m/s^2 [1]

(b) Find the distance travelled by the car between $t = 0$ and $t = 20$.

Answer ..... m [2]

(c) The distance travelled by the car between $t = 20$ and $t = k$ is 60 m.

Find the value of k.

Answer $k =$ ..... [2]

21 $y = \frac{3}{5} \times 2^x$ O/N16/21/8

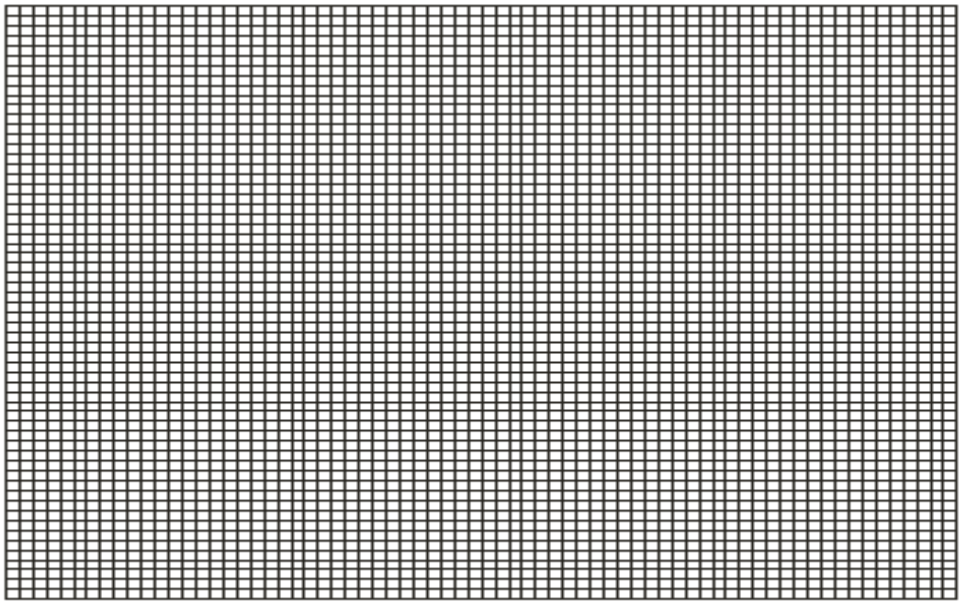
The table shows some values of x and the corresponding values of y, correct to one decimal place where necessary.

x	−1.5	−1	0	1	2	2.5	3	3.5	4
y	p	0.3	0.6	1.2	2.4	3.4	4.8	6.8	9.6

(a) Calculate p.

Answer ..... [1]

(b) On the grid,
- using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-2 \leqslant x \leqslant 4$,
- using a scale of 1 cm to 1 unit, draw a vertical y-axis for $0 \leqslant y \leqslant 10$,
- plot the points from the table and join them with a smooth curve. [3]



(c) By drawing a tangent, estimate the gradient of the curve at the point where $x = 2.5$.

Answer ..... [2]

(d) (i) On the same grid, draw the straight line that passes through $(-0.4, 0)$ and $(2, 3.6)$.

[1]

(ii) Find the equation of this line in the form $y = mx + c$.

Answer ..... [2]

(iii) Write down the x-coordinates of the points where the line intersects the curve.

Answer $x =$ ..... and $x =$ ..... [1]

(iv) These x-coordinates satisfy the equation

$$2^x = Ax + B.$$

Find the values of A and B.

Answer $A =$ ..... $B =$ ..... [2]

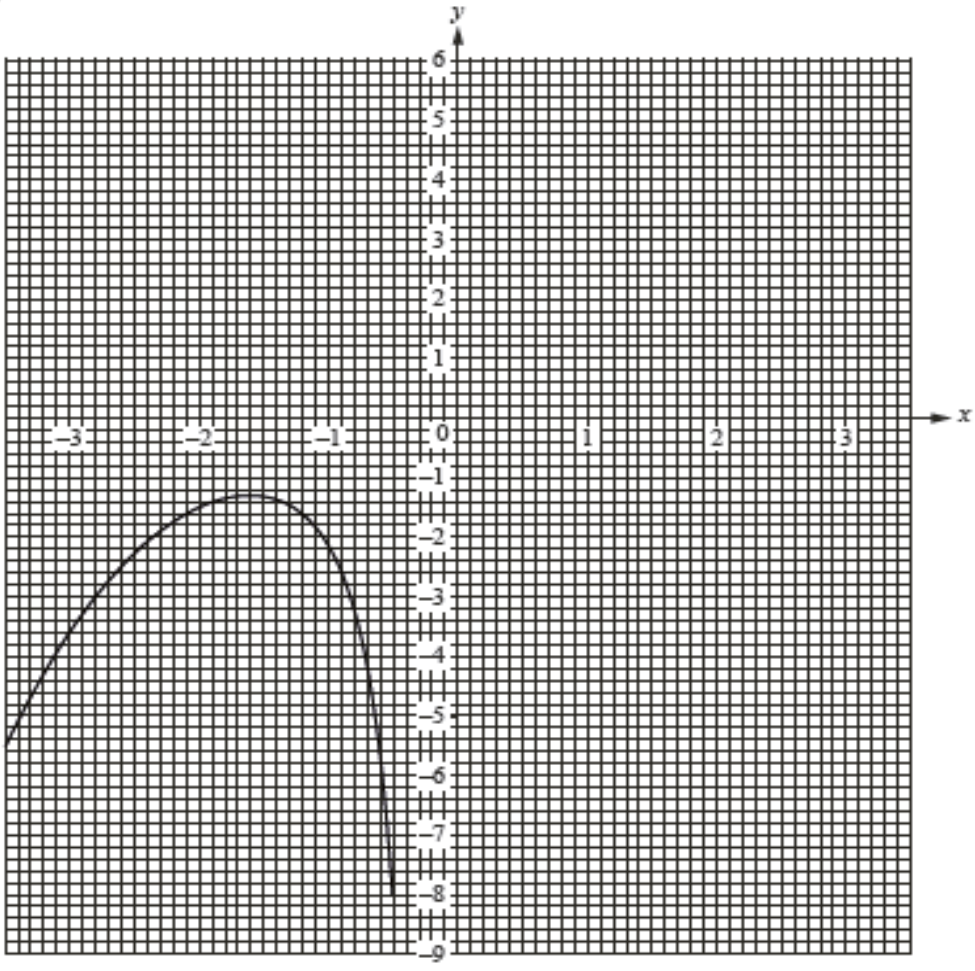
22 $y = \frac{x^3}{8} - \frac{2}{x^2}$, $x \neq 0$

(a) Complete the table of values.

x	0.5	1	1.5	2	2.5	3	3.5
y	−8.0	−1.9	−0.5	0.5	1.6		

[2]

(b)



The graph of $y = \frac{x^3}{8} - \frac{2}{x^2}$ for $-3.5 \leqslant x \leqslant -0.5$ has already been drawn.

On the grid, draw the graph of $y = \frac{x^3}{8} - \frac{2}{x^2}$ for $0.5 \leqslant x \leqslant 3.5$. [4]

(c) Use your graph to solve the equation $\frac{x^3}{8}-\frac{2}{x^2}=0$.

$x=$ ..... [1]

(d) $\frac{x^3}{8}-\frac{2}{x^2}=k$ and k is an integer.

Write down a value of k when the equation $\frac{x^3}{8}-\frac{2}{x^2}=k$ has

(i) one answer, $k=$ ..... [1]

(ii) three answers. $k=$ ..... [1]

(e) By drawing a suitable tangent, estimate the gradient of the curve where $x=-3$.

..... [3]

(f) **(i)** By drawing a suitable line on the grid, find x when $\frac{x^3}{8}-\frac{2}{x^2}=6-x$.

$x=$ ..... [3]

(ii) The equation $\frac{x^3}{8}-\frac{2}{x^2}=6-x$ can be written as $x^5+ax^3+bx^2+c=0$.

Find the values of a, b and c.

$a=$ .....

$b=$ .....

$c=$ ..... [4]

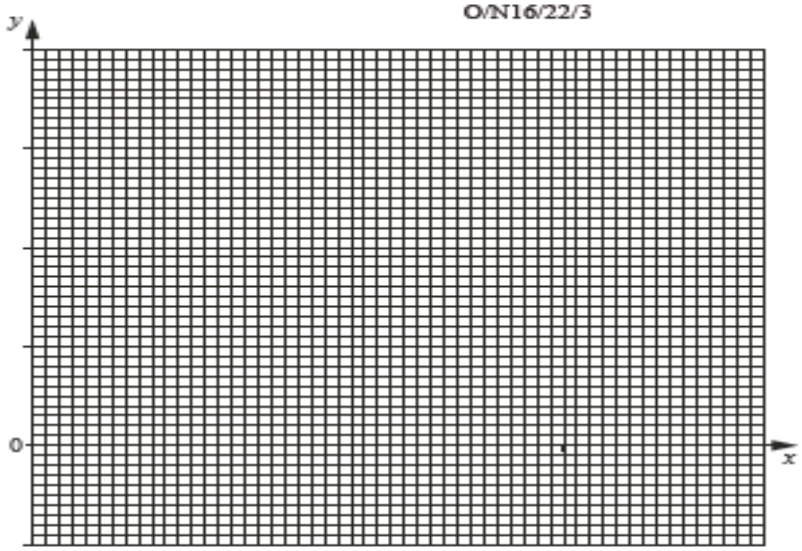
23 (a) Complete the table of values for $y = \frac{-x}{20}(x^2 - 10)$.

x	0	1	2	3	4	5
y	0	−0.45	−0.6	−0.15	1.2	

[1]

(b) Using a scale of 2 cm to 1 unit on both axes, draw the graph of $y = \frac{x}{20}(x^2 - 10)$ for $0 \leqslant x \leqslant 5$.

O/N16/22/3



[2]

(c) By drawing a tangent, estimate the gradient of the curve at the point where $x = 2.5$.

Answer ..... [2]

(d) Use your graph to solve the equation $\frac{x}{20}(x^2 - 10) = 0$ for $0 \leqslant x \leqslant 5$.

Answer $x =$ ..... or ..... [2]

(e) The graph of $y = \frac{x}{20}(x^2 - 10)$, together with the graph of a straight line L, can be used to solve the equation $x^3 + 10x - 80 = 0$ for $0 \leqslant x \leqslant 5$.

(i) Find the equation of line L.

Answer ..... [2]

(ii) Draw the graph of line L on the grid. [1]

(iii) Hence solve the equation $x^3 + 10x - 80 = 0$ for $0 \leqslant x \leqslant 5$.

Answer $x =$ ..... [1]

Graphs of functions and Travel graphs

Compiled by : Mustafa Asif

Marking Scheme

1(a)	Correct graph	**2**	**B1** for ruled line from (0, 0) to (20, 15) **B1** for ruled line from (*their* 20, 15) to (*their* 20+ 40, 15)
1(b)	0.75 oe	**1**	

2(a)	1.25 oe	**1**	
2(b)	Correct smooth curve	**2**	**B1FT** for at least 6 points correctly plotted
2(c)	$y = -\frac{1}{5}x + 2.4$ oe final answer	**3**	**M1** for $\frac{d-b}{c-a}$ from correct (a, b) and (c, d) **M1** for correct method to find y intercept
2(d)	line drawn through (1, 3) with negative gradient, crossing the curve twice	**B1**	
	5.8 to 6.2	**B1**	

3(a)	(–)2	**1**	
3(b)	20	**1**	
3(c)	600	**1**	
3(d)	40 or $10 + 30 \times \left\| \frac{\textit{their}\ \textbf{(a)}}{2} \right\|$ **ft**	**1**	
4(a)(i)	–8.5	**1**	
4(a)(ii)	8 points correctly plotted and joined with a smooth curve on correct axes	**3**	**B1** for correct scale **B1** for 6 or 7 given table points correctly plotted on *their* axes **B1** for smooth curve through all 8 points on *their* consistent axes
4(a)(iii)	2.5 – 6.5 (**dep** on tangent **soi**)	**2**	**M1** for tangent at $x = 1.5$ **soi**
4(a)(iv)	–0.85 to –0.95	**2**	**M1** for $y = 1$ **soi**

5(a)	2.4 oe final answer	**1**	
5(b)	32	**1**	
5(c)	16 nfww	**3**	**M2** for $\frac{1}{2} \times (44 + 20) \times 10$ oe $= 20k$ oe or **M1** for $\frac{1}{2} \times (44 + 20) \times 10$ oe, or for $20k$ oe = *their* distance travelled from $t = 0$ to $t = 10$

6(a)	–1.6 **oe**	**1**	
6(b)	Correct smooth curve	**3**	**B2FT** for 7 or 8 points correctly plotted or **B1FT** for 5 or 6 points correctly plotted
6(c)(i)	Tangent drawn at (–2, 0.8)	**B1**	
6(c)(ii)	–3.1 to –2.2	**B1**	Dependent on tangent drawn at $x = -2$
6(d)	–2.5 to –2.3 1.4 to 1.6 2.7 to 2.9	**3**	**FT** reading *their* graph at $y = 2$ Tolerance ± 1 mm **B1** for each one correct After 0 scored, **SC1** for $y = 2$ **soi**
7(a)	ruled line from (0, 0) to (30, 20) **and** ruled line from (30, 20) to (90, 20) **and** ruled line from (90, 20) to (110, 0)	**2**	**B1** for a graph with one error
7(b)	1700 nfww	**2**	**M1** for a correct attempt to find a relevant area under the graph, or **B1** for two of 300, 1200, 200.

8(a)	–1.6 **oe**	**1**	
8(b)	Correct smooth curve	**3**	**B2FT** for 7 or 8 points correctly plotted or **B1FT** for 5 or 6 points correctly plotted
8(c)(i)	Tangent drawn at (–2, 0.8)	**B1**	
8(c)(ii)	–3.1 to –2.2	**B1**	Dependent on tangent drawn at $x = -2$
8(d)	–2.5 to –2.3 1.4 to 1.6 2.7 to 2.9	**3**	**FT** reading *their* graph at $y = 2$ Tolerance ± 1 mm **B1** for each one correct After 0 scored, **SC1** for $y = 2$ **soi**
9(a)	BC: constant speed 18 m/s for 50 s CD: deceleration 1.2 m/s² for 15 s	**3**	**B1** for BC correct and **B2** for CD completely correct or **B1** for CD with one error or omission If 0 marks scored then **SC1** for BC is constant speed and CD is deceleration
9(b)	1215	**2**	**M1** for ½ x 18 x(50 + 85) oe or one correct area : 180 or 900 or 135 or **SC1** for answer 1080
10(a)	$\frac{u}{10}$	**1**	
10(b)	$\frac{u}{2}$	**1**	
10(c)	$55u$	**2**	**M1** for attempt to find a relevant area under the graph, soi by $50u$ or $5u$ or $60u$

11(a)	5.5, 5.5 oe	**1**	Both correct
11(b)	Correct smooth curve	**3**	**B2FT** for 8 or 9 points correctly plotted or **B1FT** for 6 or 7 points correctly plotted
11(c)	tangent drawn at $x = 1.5$	**B1**	Dependent on a curve drawn between $x = 1$ and $x = 2$
	–1.7 to –1.3	**B1**	
11(d)	$x \leqslant$ 0.6 to 0.9 $x \geqslant$ 5.1 to 5.4	**2**	**B1** for one correct or **SC1** for answers reversed
11(e)(i)	Ruled line passing through (0, 3) and (4, 0) crossing curve twice	**2**	**B1** for short or unruled line or for two correct points plotted
11(e)(ii)	$A = -9$, $B = -4$	**2**	**B1** for either correct or $2x^2 - 9x - 4$ [=0] or **M1** for $\left(\frac{x^2}{2} - 3x + 2\right) = \frac{12 - 3x}{4}$ oe After 0, **SC1** for $A = -9.2$ to -8.8 and $B = -4.2$ to -3.8
12(a)	$\frac{v}{10}$ oe	**1**	
12(b)	20 nfww	**3**	**M1** for $\frac{1}{2} \times (40 + 80) \times v$ oe or **B1** for two of $15v$, $40v$, $5v$. **M1** for *their* $60v$ = *their*(1200)
13(a)	21	**1**	
13(b)	$\frac{18}{20}$ oe	**1**	
13(c)	420	**2**	**M1** for a correct, complete, method to find the area. e.g. $\frac{1}{2} \times (30 + 12) \times (60 - 40)$; $12 \times (60 - 40) + \frac{1}{2} \times (60 - 40) \times (30 - 12)$; $(60 - 40) \times 30 - \frac{1}{2} \times (60 - 40) \times (30 - 12)$

14(a)(ii)	$y = -2x - 1$ oe **FT** *their* gradient from (a)(i)	**2**	**B1** for answer $y =$ *their* $(-2)x + k$, where $k \neq$ *their* 5 or **M1** for $3 =$ *their* $(-2) \times -2 + k$ oe
14(b)(i)	3.5	**1**	
14(b)(ii)	Correct smooth curve through 8 correct points	**3**	**B2FT** for 7 or 8 points correctly plotted or **B1FT** for 5 or 6 points correctly plotted
14(b)(iii)	Clear correct tangent drawn at (1, 1)	**M1**	
	–2.4 to –1.6	**A1**	
14(b)(iv)	0.6 to 0.8 and 4.2 to 4.4	**2**	**FT** reading from *their* graph at $y = 2$ **B1** for one correct or for $y = 2$ soi

15(a)(i)	–4.5 –4.5	**1**	Both correct
15(a)(ii)	Correct smooth curve	**3FT**	**B2FT** for 8 or 9 points correctly plotted Or **B1FT** for 6 or 7 points correctly plotted Or **B1** for the correct scales drawn
15(a)(iii)	–2.4 to –1.6 dependent on tangent drawn	**2**	Accept a correctly formed $\Delta y \div \Delta x$ isw **B1** for tangent drawn at (3, 1.5)
15(a)(iv)(a)	–2 **cao**		
15(a)(iv)(b)	–2.4 to –2.3 and 4.3 to 4.4		**FT** reading their graph at $y =$ *their* –2 Tolerance ± 1 small square **B1 FT** for one correct
15(b)(i)	4	**1**	
15(b)(ii)	3	**1**	
15(b)(iii)	324	**1**	

16(a)	11	**1**	
16(b)	30	**1**	
16(c)(i)	line joining (1125, 25) to (1155, 0)	**1**	

17(a)	18	**2**	**M1** for $\frac{v-12}{15}$ or $\frac{12-v}{15}$ oe
17(b)	345	**2**	**B1FT** for a correct partial area: 120 or 225 or 300 or 45 or 180 or **M1FT** for $12 \times 25 + 0.5 \times 15 \times$ (*their*18 – 12) oe

18(a)	17	**1**	
18(b)	Smooth curve through 7 correct points	**3**	Mark the curve first **B2** for at least 5 ft plots correct **B1** for at least 4 ft plots correct
18(c)	–1.7 to –1.4, –0.5 to –0.2, 1.7 to 2.0	**2**	FT **B1** for 2 correct
18(d)	3 to 5 with tangent drawn	**2**	**B1** for ruled solid tangent drawn
18(e)(i)	Correct ruled line drawn	**1**	
18(e)(ii)	$a = 7$, $b = 4$	**2**	**B1** for one correct or $a = 6.8$ to 7.2 and $b = 3.8$ to 4.2
18(e)(iii)	–2.4 to –2.1 or –0.7 to –0.5	**1**	FT
19 (a)	10.4 or any equivalent	2 *	**M1** for $\frac{v-4}{8}=\frac{8}{10}$ oe or **B1** for 6.4 oe; or for 1.6 oe; seen
(b)	80	2 *	**C1** for 140 or **M1** for $10 \times (4 + 12)/2$ oe
(c)	Curve, concave upwards, from (0, 0) to (10, their(b)	1 √	independent
	Straight line from (10, their(b)) to (15, 60 + their(b))	1 √	independent

20 (a)	(–)0.9 oe	1	
(b)	420	2*	**M1** for $\frac{1}{2}\times 20\times(12+30)$ oe
(c)	25	2*	**M1** for $(k-20)\times 12=60$ oe or **C1** for $k=5$

21 (a)	0.2 or 0.21[2…]	**1**	
(b)	Correct axes	**B1**	
	Correct shape curve through 9 correct points	**B2**	**B1ft** for at least 7 correct points plotted
(c)	Clear, correct, tangent drawn	**M1**	
	2.2 to 2.5	**A1**	
(d) (i)	Ruled line from (–0.4, 0) to (2, 3.6)	**1**	
(ii)	$y=1.5x+0.6$ or $y=\frac{3}{2}x+\frac{3}{5}$	**2**	**B1** for $m=1.5$ oe or for $c=0.6$ oe or for correct equation in a different form
(iii)	0 **and** 3.1 to 3.2	**1ft**	FT intersections of *their* ruled line with *their* curve
(iv)	$A=2.4$ to 2.6	**1**	
	$B=1$	**1**	

22(a)	3.2 or 3.15 or 3.152 to 3.153 5.2 or 5.19 or 5.20 or 5.196…	**2**	**B1** for each
22(b)	Correct graph for $0.5\leqslant x\leqslant 3.5$	**4**	**B3FT** for 6 or 7 correct points or **B2FT** for 4 or 5 correct points or **B1FT** for 2 or 3 correct points
22(c)	1.7 to 1.8	**1FT**	**FT** *their* graph if one answer
22(d)(i)	Any integer $k\geqslant -1$	**1**	
22(d)(ii)	Any integer $k<-1$	**1**	
22(e)	Tangent ruled at $x=-3$	**B1**	
	2.5 to 4	**B2**	**dep** on tangent drawn at $x=-3$ or close attempt at tangent at $x=-3$ **M1** for rise/run also dep on tangent at $x=-3$ or close attempt at tangent at $x=-3$

22(f)(i)	$y = 6 - x$ ruled accurately	**M2**	**M1** for correct line but freehand or ruled line gradient –1.1 to –0.9, or through (0, 6) but not $y = 6$
	$2.85 \leqslant x \leqslant 3$	**A1**	
22(f)(ii)	$[a =]$ 8 $[b =]$ –48 $[c =]$ –16	**4**	**B3** for 2 correct or $x^5 + 8x^3 - 48x^2 - 16 = 0$ seen or $-x^5 - 8x^3 + 48x^2 + 16 = 0$ seen or **M2** for correct multiplication by $8x^2$ or **B1** for answers $\pm 8, \pm 48, \pm 16$ or **M1** for $\frac{x^2 \times x^3 - 8 \times 2}{x^2 \times 8} = 6 - x$ or **M1** for correct multiplication by 8 or **M1** for correct multiplication by x^2

23	**(a)**	3.75	**1**	
	(b)	Correct curve ft	**2ft**	B1 for 4 correct plots ft
	(c)	(0.3 to 0.5) ft	**2ft**	M1 for a reasonable tangent at $x = 2.5$
	(d)	0 cao (3.05 to 3.25) ft	**2ft**	B1 for either
	(e) (i)	$y = 4 - x$	**2**	M1 for $x^3 + 10x - 80 = 0 \equiv \frac{x}{20}(x^2 - 10) = ax + b$ oe
	(ii)	L drawn on the grid ft	**1ft**	Dependent on at least 1 mark in (e)(i).
	(iii)	(3.55) ft	**1ft**	Dependent on at least 1 mark in (e)(i).